1. Let \( f \) be a differentiable function from \( \mathbb{R} \) to \( \mathbb{R} \) such that \( |f(x) - f(y)| \leq 2|x - y|^{3/2} \), for all \( x, y \in \mathbb{R} \). If \( f(0) = 1 \) then \( \int_0^1 f^2(x) \, dx \) is equal to

(1) 0 (2) \( \frac{1}{2} \) (3) 2 (4) 1

Ans. (4)

Sol. \[ |f(x) - f(y)| < 2|x - y|^{3/2} \]

divide both sides by \( |x - y|^{1/2} \)

\[ \frac{f(x) - f(y)}{x - y} \leq 2|x - y| \]

apply limit \( x \to y \)

\[ |f'(y)| < 0 \]

\( \Rightarrow \) \( f'(y) = 0 \)

\( \Rightarrow \) \( f(y) = c \)

\( \Rightarrow \) \( f(x) = 1 \)

\[ \int_0^1 f^2(x) \, dx = 1 \]

2. If \[ \int_0^{\frac{\pi}{2}} \frac{\tan \theta}{\sqrt{2k \sec \theta}} \, d\theta = 1 - \frac{1}{\sqrt{2}} \] \((k > 0)\), then the value of \( k \) is:

(1) 2 (2) \( \frac{1}{2} \) (3) 4 (4) 1

Ans. (1)

Sol. \[ \frac{1}{\sqrt{2k}} \int_0^{\frac{\pi}{2}} \frac{\tan \theta}{\sqrt{\sec \theta}} \, d\theta = \frac{1}{\sqrt{2k}} \int_0^{\frac{\pi}{2}} \frac{\sin \theta}{\cos \theta} \, d\theta = -\frac{\sqrt{2}}{2k} \left( \frac{1}{\sqrt{2}} - 1 \right) \]

given it is \( 1 - \frac{1}{\sqrt{2}} \) \( \Rightarrow \) \( k = 2 \)

3. The coefficient of \( t^4 \) in the expansion of \( \left( \frac{1-t^6}{1-t} \right)^3 \) is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

Sol. \( (1-t^6)^3 (1-t)^{-3} \)

\( (1-t^6) - 3t^6 + 3t^{12} (1-t)^{-3} \)

\[ \Rightarrow \] coefficient of \( t^4 \) in \( (1-t)^{-3} \) is

\[ 3^{4+1}C_4 = 6C_2 = 15 \]

4. For each \( x \in \mathbb{R} \), let \([x]\) be the greatest integer less than or equal to \( x \). Then

\[ \lim_{x \to 0^+} x \left( \frac{[x] + [x]}{x} \right) \sin \frac{\pi}{x} \]

is equal to

(1) \(-\sin 1\) (2) 0 (3) 1 (4) \( \sin 1\)

Ans. (1)

Sol. \[ \lim_{x \to 0^+} x \left( \frac{[x] + [x]}{x} \right) \sin \frac{\pi}{x} = -\sin 1 \]

5. If both the roots of the quadratic equation \( x^2 - mx + 4 = 0 \) are real and distinct and they lie in the interval \([1, 5]\), then \( m \) lies in the interval:

(1) \((4, 5)\) (2) \((3, 4)\) (3) \((5, 6)\) (4) \((-5, -4)\)

Ans. (Bonus/1)

Sol. \( x^2 - mx + 4 = 0 \)

\( \alpha, \beta \in [1, 5] \)

(1) \( D > 0 \) \( \Rightarrow \) \( m^2 - 16 > 0 \)

\( \Rightarrow m \in (-\infty, -4) \cup (4, \infty) \)

(2) \( f(1) \geq 0 \) \( \Rightarrow \) \( 5 - m \geq 0 \) \( \Rightarrow m \in (-\infty, 5] \)

(3) \( f(5) \geq 0 \) \( \Rightarrow \) \( 29 - 5m \geq 0 \) \( \Rightarrow m \in \left(-\infty, \frac{29}{5}\right] \)

(4) \( 1 < \frac{-b}{2a} < 5 \) \( \Rightarrow 1 < \frac{m}{2} < 5 \) \( \Rightarrow m \in (2, 10) \)

\( \Rightarrow m \in (4, 5) \)

No option correct : Bonus

* If we consider \( \alpha, \beta \in (1, 5) \) then option (1) is correct.
6. If 
\[ A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ 2e^{-t} \sin t & -2e^{-t} \cos t & 1 \end{bmatrix} \]

Then A is:

(1) Invertible only if \( t = \frac{\pi}{2} \)

(2) not invertible for any \( t \in \mathbb{R} \)

(3) invertible for all \( t \in \mathbb{R} \)

(4) invertible only if \( t = \pi \)

Ans. (3)

Sol.
\[ |A| = e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 2\sin t & 2\cos t & 1 \end{vmatrix} \]
\[ = e^{-t} [5 \cos^2 t + 5 \sin^2 t] \quad \forall \ t \in \mathbb{R} \]
\[ = 5e^{-t} \neq 0 \quad \forall \ t \in \mathbb{R} \]

7. The area of the region
\[ A = \left\{ (x, y): 0 \leq y \leq \sqrt{x} + 1 \text{ and } -1 \leq x \leq 1 \right\} \]
in sq. units, is:

(1) \( \frac{2}{3} \)  
(2) \( \frac{1}{3} \)  
(3) 2  
(4) \( \frac{4}{3} \)

Ans. (3)

Sol. The graph is as follows

[Diagram]

\[ \int_{-1}^{1} (\sqrt{x} + 1) \, dx = 2 \]

8. Let \( z_0 \) be a root of the quadratic equation, \( x^2 + x + 1 = 0 \). If \( z = 3 + 6iz_0^8 - 3iz_0^{10} \), then \( \arg z \) is equal to:

(1) \( \frac{\pi}{4} \)  
(2) \( \frac{\pi}{3} \)  
(3) 0  
(4) \( \frac{\pi}{6} \)

Ans. (1)

Sol. \( z_0 = \omega \) or \( \omega^2 \) (where \( \omega \) is a non-real cube root of unity)
\[ z = 3 + 6i(\omega^8) - 3i(\omega^{10}) \]
\[ = 3 + 3i \]
\[ \Rightarrow \arg z = \frac{\pi}{4} \]

9. Let \( \vec{a} = \hat{i} + \hat{j} + \sqrt{2k}, \vec{b} = b_1 \hat{i} + b_2 \hat{j} + \sqrt{2k} \) and \( \vec{c} = 5\hat{i} + \hat{j} + \sqrt{2k} \) be three vectors such that the projection vector of \( \vec{b} \) on \( \vec{a} \) is \( \vec{a} \). If \( \vec{a} + \vec{b} \) is perpendicular to \( \vec{c} \), then \( |\vec{b}| \) is equal to:

(1) \( \sqrt{22} \)  
(2) 4  
(3) \( \sqrt{32} \)  
(4) 6

Ans. (4)

Sol. Projection of \( \vec{b} \) on \( \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = |\vec{a}| \)
\[ \Rightarrow b_1 + b_2 = 2 \quad \ldots (1) \]
and \( (\vec{a} + \vec{b}) \perp \vec{c} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{c} = 0 \)
\[ \Rightarrow 5b_1 + b_2 = -10 \quad \ldots (2) \]
from (1) and (2) \( b_1 = -3 \) and \( b_2 = 5 \)
then \( |\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6 \)

10. Let \( A(4, -4) \) and \( B(9, 6) \) be points on the parabola, \( y^2 + 4x \). Let \( C \) be chosen on the arc \( AOB \) of the parabola, where \( O \) is the origin, such that the area of \( \Delta ACB \) is maximum. Then, the area (in sq. units) of \( \Delta ACB \), is:

(1) \( 31 \frac{3}{4} \)  
(2) 32  
(3) \( 30 \frac{1}{2} \)  
(4) \( 31 \frac{1}{4} \)

Ans. (4)
11. The logical statement
\[ \neg (\neg p \lor q) \lor (p \land r) \land (\neg q \land r) \] is equivalent to:
(1) \((p \land r) \land \neg q\)
(2) \((\neg p \land \neg q) \land r\)
(3) \(\neg p \lor r\)
(4) \((p \land \neg q) \lor r\)
Ans. (1)

Sol.
\[ s[\neg (\neg p \lor q) \land (p \land r)] \land (\neg q \land r) \]
\[ = [(p \land \neg q) \lor (p \land r)] \land (\neg q \land r) \]
\[ = p \land (\neg q \land r) \]
\[ = (p \land r) \land \neg q \]

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is:
(1) \(\frac{26}{49}\)
(2) \(\frac{32}{49}\)
(3) \(\frac{27}{49}\)
(4) \(\frac{21}{49}\)
Ans. (2)

Sol. \( E_1 \) : Event of drawing a Red ball and placing a green ball in the bag
\( E_2 \) : Event of drawing a green ball and placing a red ball in the bag
\( E \) : Event of drawing a red ball in second draw

\[ P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right) \]
\[ = \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49} \]

13. If \(0 \leq x < \frac{\pi}{2}\), then the number of values of \(x\) for which \(\sin x - \sin 2x + \sin 3x = 0\), is:
(1) 2
(2) 1
(3) 3
(4) 4
Ans. (1)

Sol. \(\sin x - \sin 2x + \sin 3x = 0\)
\[ \Rightarrow (\sin x + \sin 3x) - \sin 2x = 0 \]
\[ \Rightarrow 2\sin x \cos x - \sin 2x = 0 \]
\[ \Rightarrow \sin 2x(2 \cos x - 1) = 0 \]
\[ \Rightarrow \sin 2x = 0 \text{ or } \cos x = \frac{1}{2} \]
\[ \Rightarrow x = 0, \frac{\pi}{3} \]

14. The equation of the plane containing the straight line \(\frac{x}{2} = \frac{y}{3} = \frac{z}{4}\) and perpendicular to the plane containing the straight lines
\[ \frac{x}{3} = \frac{y}{4} = \frac{z}{2} \text{ and } \frac{x}{4} = \frac{y}{2} = \frac{z}{3} \] is:
(1) \(x + 2y - 2z = 0\)
(2) \(x - 2y + z = 0\)
(3) \(5x + 2y - 4z = 0\)
(4) \(3x + 2y - 3z = 0\)
Ans. (2)
Vector along the normal to the plane containing the lines
\[ \frac{x}{3} - \frac{y}{4} = \frac{z}{2} \] and \[ \frac{x}{4} - \frac{y}{2} = \frac{z}{3} \]
is \( 8\hat{i} - \hat{j} - 10\hat{k} \)
vector perpendicular to the vectors \( 2\hat{i} + 3\hat{j} + 4\hat{k} \)
and \( 8\hat{i} - \hat{j} - 10\hat{k} \) is \( 26\hat{i} - 52\hat{j} + 26\hat{k} \)
so, required plane is
\[ 26x - 52y + 26z = 0 \]
\[ x - 2y + z = 0 \]

15. Let the equations of two sides of a triangle be 3x – 2y + 6 = 0 and 4x + 5y – 20 = 0. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :
(1) 122y – 26x – 1675 = 0
(2) 26x + 61y + 1675 = 0
(3) 122y + 26x + 1675 = 0
(4) 26x – 122y – 1675 = 0
Ans. (4)

16. If \( x = 3 \tan t \) and \( y = 3 \sec t \), then the value of \( \frac{dy}{dx} \) at \( t = \frac{\pi}{4} \), is:
(1) \( \frac{3}{2\sqrt{2}} \)
(2) \( \frac{1}{3\sqrt{2}} \)
(3) \( \frac{1}{6} \)
(4) \( \frac{1}{6\sqrt{2}} \)
Ans. (4)

17. If \( x = \sin^{-1}(\sin 10) \) and \( y = \cos^{-1}(\cos 10) \), then \( y \approx x \) is equal to:
(1) \( \pi \)
(2) \( 7\pi \)
(3) \( 0 \)
(4) \( 10 \)
Ans. (1)

18. If the lines \( x = ay+b \), \( z = cy+d \) and \( x=a'z + b' \), \( y = c'z + d' \) are perpendicular, then:
(1) \( cc' + a + a' = 0 \)
(2) \( aa' + c + c' = 0 \)
(3) \( ab' + bc' + 1 = 0 \)
(4) \( bb' + cc' + 1 = 0 \)
Ans. (2)

19. The number of all possible positive integral values of \( \alpha \) for which the roots of the quadratic equation, \( 6x^2 – 11x + \alpha = 0 \) are rational numbers is :
(1) \( 2 \)
(2) \( 5 \)
(3) \( 3 \)
(4) \( 4 \)
Ans. (3)

Given both the lines are perpendicular
\( \Rightarrow a' + c' + c = 0 \)

19. The number of all possible positive integral values of \( \alpha \) for which the roots of the quadratic equation, \( 6x^2 – 11x + \alpha = 0 \) are rational numbers is :
(1) \( 2 \)
(2) \( 5 \)
(3) \( 3 \)
(4) \( 4 \)
Ans. (3)

"
20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is:

(1) \( \frac{2}{\sqrt{3}} \)  
(2) \( \frac{3}{2} \)  
(3) \( \sqrt{3} \)  
(4) 2

Ans. (1)

Sol.

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

\[ 2a = 4 \quad a = 2 \]

\[ \frac{x^2}{4} - \frac{y^2}{b^2} = 1 \]

Passes through (4,2)

\[ 4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3} \Rightarrow e = \frac{2}{\sqrt{3}} \]

21. Let \( A = \{ x \in \mathbb{R} : x \text{ is not a positive integer} \} \)

Define a function \( f : A \rightarrow \mathbb{R} \) as \( f(x) = \frac{2x}{x-1} \) then \( f \) is

(1) injective but not surjective
(2) not injective
(3) surjective but not injective
(4) neither injective nor surjective

Ans. (1)

Sol.

\[ f(x) = 2 \left( 1 + \frac{1}{x-1} \right) \]

\[ f'(x) = -\frac{2}{(x-1)^2} \]

\[ \Rightarrow f \text{ is one-one but not onto} \]

22. If \( f(x) = \int \frac{5x^8 + 7x^6}{x^2 + 1 + 2x^7} \, dx \), then the value of \( f(1) \) is:

(1) \( -\frac{1}{2} \)  
(2) \( \frac{1}{2} \)  
(3) \( -\frac{1}{4} \)  
(4) \( \frac{1}{4} \)

Ans. (4)

Sol.

\[ \int \frac{5x^8 + 7x^6}{x^2 + 1 + 2x^7} \, dx = \frac{1}{2} \cdot \frac{1}{x^5} + \frac{1}{x^3} + C \]

As \( f(0) = 0 \), \( f(x) = \frac{x^7}{2x^7 + x^2 + 1} \)

\[ f(1) = \frac{1}{4} \]

23. If the circles \( x^2 + y^2 - 16x - 20y + 164 = r^2 \) and \( (x-4)^2 + (y-7)^2 = 36 \) intersect at two distinct points, then:

(1) \( 0 < r < 1 \)
(2) \( 1 < r < 11 \)
(3) \( r > 11 \)
(4) \( r = 11 \)

Ans. (2)

Sol. \( x^2 + y^2 - 16x - 20y + 164 = r^2 \)

Let \( A(8,10), R_1 = r \)

\( (x-4)^2 + (y-7)^2 = 36 \)

\( B(4,7), R_2 = 6 \)

\[ |R_1 - R_2| < AB < R_1 + R_2 \]

\( \Rightarrow 1 < r < 11 \)

24. Let \( S \) be the set of all triangles in the \( xy \)-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in \( S \) has area 50 sq. units, then the number of elements in the set \( S \) is:

(1) 9  
(2) 18  
(3) 32  
(4) 36

Ans. (4)

Sol. Let \( A(\alpha,0) \) and \( B(0,\beta) \)

be the vectors of the given triangle AOB

\( \Rightarrow |\alpha| = 100 \)

\( \Rightarrow \) Number of triangles

\( = 4 \times (\text{number of divisors of 100}) \)

\( = 4 \times 9 = 36 \)

25. The sum of the following series

\[ 1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} \]

\[ + \frac{15(1^2 + 2^2 + \ldots + 5^2)}{11} + \ldots \] up to 15 terms, is:

(1) 7820  
(2) 7830  
(3) 7520  
(4) 7510

Ans. (1)
### 26. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then \( \frac{a}{c} \) is equal to:

\[
\begin{align*}
(1) & \quad \frac{1}{2} & (2) & \quad 4 \\
(3) & \quad 2 & (4) & \quad \frac{7}{13}
\end{align*}
\]

**Ans. (2)**

**Sol.**

\[
a = A + 6d \\
b = A + 10d \\
c = A + 12d
\]

a, b, c are in G.P.

\( \Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d) \)

\[
\Rightarrow \frac{A}{d} = -14
\]

\[
\begin{align*}
\frac{a}{c} & = \frac{A + 6d}{A + 12d} \\
& = \frac{6 + \frac{A}{d}}{12 + \frac{A}{d}} \\
& = \frac{6 - 14}{12 - 14} = 4
\end{align*}
\]

### 27. If the system of linear equations

\[
\begin{align*}
x & = -4y + 7z = g \\
3y & = 5z = h \\
-2x & = 5y - 9z = k
\end{align*}
\]

is consistent, then:

\( (1) g + h + k = 0 \)

\( (2) 2g + h + k = 0 \)

\( (3) g + h + 2k = 0 \)

\( (4) g + 2h + k = 0 \)

**Ans. (2)**

**Sol.**

\[
\begin{align*}
P_1 & = x - 4y + 7z - g = 0 \\
P_2 & = 3x - 5y - h = 0 \\
P_3 & = -2x + 5y - 9z - k = 0
\end{align*}
\]

Here \( \Delta = 0 \)

\( 2P_1 + P_2 + P_3 = 0 \) when \( 2g + h + k = 0 \)

### 28. Let \( f: [0,1] \rightarrow \mathbb{R} \) be such that \( f(xy) = f(x)f(y) \) for all \( x, y, \in [0,1] \), and \( f(0) \neq 0 \). If \( y = y(x) \) satisfies the differential equation, \( \frac{dy}{dx} = f(x) \) with \( y(0) = 1 \), then \( y \left( \frac{1}{4} \right) + y \left( \frac{3}{4} \right) \) is equal to:

\( (1) 4 \quad (2) 3 \quad (3) 5 \quad (4) 2 \)

**Ans. (2)**

**Sol.**

\[
\begin{align*}
f(xy) & = f(x)f(y) \\
f(0) & = 1 \text{ as } f(0) \neq 0 \\
\Rightarrow f(x) & = 1 \\
\frac{dy}{dx} & = f(x) = 1 \\
\Rightarrow y & = x + c \\
\text{At, } x = 0, y = 1 & \Rightarrow c = 1 \\
y & = x + 1 \\
\Rightarrow y \left( \frac{1}{4} \right) + y \left( \frac{3}{4} \right) & = \frac{1}{4} + \frac{3}{4} + 1 = 3
\end{align*}
\]

### 29. A data consists of \( n \) observations:

\( x_1, x_2, \ldots, x_n \). If \( \sum_{i=1}^{n} (x_i + 1)^2 = 9n \) and \( \sum_{i=1}^{n} (x_i - 1)^2 = 5n \), then the standard deviation of this data is:

\( (1) 5 \quad (2) \sqrt{5} \quad (3) \sqrt{7} \quad (4) 2 \)

**Ans. (2)**

**Sol.**

\[
\begin{align*}
a & = A + 6d \\
a & = A + 12d \\
\Rightarrow \frac{A}{d} & = -14
\end{align*}
\]

\[
\begin{align*}
a & = 6 + \frac{A}{d} \\
& = \frac{6 - 14}{12 - 14} = 4
\end{align*}
\]
Sol. \( \sum (x_i + 1)^2 = 9n \) \( \ldots (1) \)
\( \sum (x_i - 1)^2 = 5n \) \( \ldots (2) \)

(1) + (2) \( \Rightarrow \sum (x_i^2 + 1) = 7n \)

\[ \frac{\sum x_i^2}{n} = 6 \]

(1) - (2) \( \Rightarrow 4\sum x_i = 4n \)

\[ \sum x_i = n \]

\[ \frac{\sum x_i^2}{n} = 1 \]

\( \Rightarrow \) variance = \( 6 - 1 = 5 \)

\( \Rightarrow \) Standard deviation = \( \sqrt{5} \)

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to:

(1) 250  (2) 374  (3) 372  (4) 375

Ans. (2)

Sol.

\[ \begin{array}{ccc}
 a_1 & a_2 & a_3 \\
 a_4 & a_5 & a_6 \\
 \end{array} \]

Number of numbers = \( 5^3 - 1 \)

\[ \begin{array}{ccc}
 a_3 & a_1 & a_2 \\
 a_4 & a_5 & a_6 \\
 \end{array} \]

2 ways for \( a_4 \)

Number of numbers = \( 2 \times 5^3 \)

Required number = \( 5^3 + 2 \times 5^3 - 1 \)

= 374