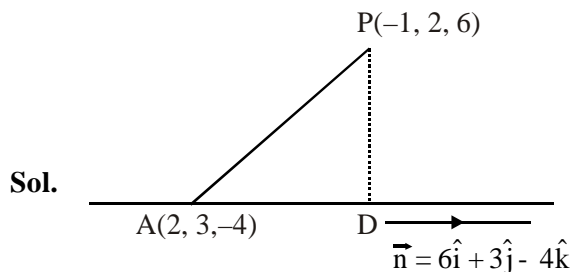


FINAL JEE-MAIN EXAMINATION – APRIL, 2019**(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM****MATHEMATICS**

1. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point $(2, 3, -4)$ and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is :

- (1) 7 (2) $4\sqrt{3}$
 (3) $2\sqrt{13}$ (4) 6

Official Ans. by NTA (1)

$$AD = \frac{|\vec{AP} \cdot \vec{n}|}{|\vec{n}|} = \sqrt{61}$$

$$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$$

2. If both the mean and the standard deviation of 50 observations x_1, x_2, \dots, x_{50} are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \dots, (x_{50} - 4)^2$ is :

- (1) 525 (2) 380
 (3) 480 (4) 400

Official Ans. by NTA (4)

Sol. Mean $(\mu) = \frac{\sum x_i}{50} = 16$

$$\text{standard deviation } (\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$$

$$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$$

 \Rightarrow New mean

$$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8 \sum x_i}{50}$$

$$= (256) \times 2 + 16 - 8 \times 16 = 400$$

TEST PAPER WITH ANSWER & SOLUTION

3. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

- (1) $(2, 0, 1)$ (2) $(4, 0, -1)$
 (3) $(-1, 0, 4)$ (4) $(1, 0, 2)$

Official Ans. by NTA (1)

- Sol. Let point P on the line is $(2\lambda + 1, -\lambda - 1, \lambda)$ foot of perpendicular Q is given by

$$\frac{x - 2\lambda - 1}{1} = \frac{y + \lambda + 1}{1} = \frac{z - \lambda}{1} = \frac{-(2\lambda - 3)}{3}$$

$$\because Q \text{ lies on } x + y + z = 3 \text{ \& } x - y + z = 3$$

$$\Rightarrow x + z = 3 \text{ \& } y = 0$$

$$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda + 3}{3} \Rightarrow \lambda = 0$$

$$\Rightarrow Q \text{ is } (2, 0, 1)$$

4. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point $P(2, 2)$ meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

- (1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{68}{15}$ (4) $\frac{34}{15}$

Official Ans. by NTA (3)

Sol. $3x^2 + 5y^2 = 32$

$$\left. \frac{dy}{dx} \right|_{(2,2)} = -\frac{3}{5}$$

$$\text{Tangent : } y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$$

$$\text{Normal : } y - 2 = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$$

$$\text{Area is } = \frac{1}{2}(\text{QR}) \times 2 = \text{QR} = \frac{68}{15}$$

5. Let λ be a real number for which the system of linear equations

$$x + y + z = 6$$

$$4x + \lambda y - \lambda z = \lambda - 2$$

$$3x + 2y - 4z = -5$$

has infinitely many solutions. Then λ is a root of the quadratic equation.

$$(1) \lambda^2 - 3\lambda - 4 = 0 \quad (2) \lambda^2 - \lambda - 6 = 0$$

$$(3) \lambda^2 + 3\lambda - 4 = 0 \quad (4) \lambda^2 + \lambda - 6 = 0$$

Official Ans. by NTA (2)

Sol. $D = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

6. The smallest natural number n , such that the

coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$

is ${}^n C_{23}$, is :

$$(1) 35 \quad (2) 38$$

$$(3) 23 \quad (4) 58$$

Official Ans. by NTA (2)

Sol. $T_r = \sum_{r=0}^n {}^n C_r x^{2n-2r} \cdot x^{-3r}$

$$2n - 5r = 1 \Rightarrow 2n = 5r + 1$$

$$\text{for } r = 15, n = 38$$

smallest value of n is 38.

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm³/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

$$(1) \frac{1}{9\pi} \quad (2) \frac{5}{6\pi} \quad (3) \frac{1}{18\pi} \quad (4) \frac{1}{36\pi}$$

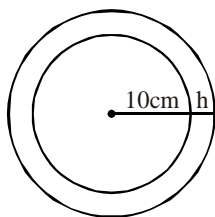
Official Ans. by NTA (3)

Sol. $V = \frac{4}{3}\pi((10+h)^3 - 10^3)$

$$\frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$$

$$-50 = 4\pi(10+5)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{1}{18} \text{ cm/min}$$



8. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

$$(1) \left(-\frac{5}{3}, 0\right) \quad (2) (5, 0)$$

$$(3) (-5, 0) \quad (4) \left(\frac{5}{3}, 0\right)$$

Official Ans. by NTA (3)

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$$a = 3, b = 4 \text{ \& } e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$$

corresponding focus will be $(-ae, 0)$ i.e., $(-5, 0)$.

9. The sum $1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + \dots$

$$+ \frac{1^3 + 2^3 + 3^3 + \dots + 15^3}{1+2+3+\dots+15} - \frac{1}{2}(1+2+3+\dots+15)]$$

$$(1) 1240$$

$$(2) 1860$$

$$(3) 660$$

$$(4) 620$$

Official Ans. by NTA (4)

Sol. $\text{Sum} = \sum_{n=1}^{15} \frac{1^3 + 2^3 + \dots + n^3}{1+2+\dots+n} - \frac{1}{2} \cdot \frac{15 \cdot 16}{2}$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2 - (n-1))}{6} - 60$$

$$= \frac{15 \cdot 16 \cdot 17}{6} - 60 = 620$$

10. If the line $ax + y = c$, touches both the curves

$x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to :

- (1) $1/2$ (2) 2
 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Official Ans. by NTA (3)

Sol. Tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$$\Rightarrow \left| \frac{\sqrt{2}/m}{\sqrt{1+m^2}} \right| = 1 \Rightarrow m = \pm 1$$

\Rightarrow Tangent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$
 compare with $y = -ax + C$

$$\Rightarrow a = \pm 1 \text{ \& } C = \pm\sqrt{2}$$

11. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \leq x \leq 1$, $-2 \leq y \leq 2$, $x \leq \frac{y}{2}$,

then for all x, y , $4x^2 - 4xy \cos \alpha + y^2$ is equal to

- (1) $4 \sin^2 \alpha - 2x^2y^2$ (2) $4 \cos^2 \alpha + 2x^2y^2$
 (3) $4 \sin^2 \alpha$ (4) $2 \sin^2 \alpha$

Official Ans. by NTA (3)

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos \alpha$$

$$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\Rightarrow \left(\cos \alpha - \frac{xy}{2} \right)^2 = (1-x^2) \left(1 - \frac{y^2}{4} \right)$$

$$x^2 + \frac{y^2}{4} - xy \cos \alpha = 1 - \cos^2 \alpha = \sin^2 \alpha$$

12. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to :

- (1) $-\frac{5}{2}$ (2) 1
 (3) $-\frac{1}{2}$ (4) -1

Official Ans. by NTA (1)

Sol. Let $x^2 = t$ $2x dx = dt$

$$\Rightarrow \frac{1}{2} \int t^2 \cdot e^{-t} dt = \frac{1}{2} \left[-t^2 \cdot e^{-t} + \int 2t \cdot e^{-t} dt \right]$$

$$= \frac{1}{2} \left(-t^2 \cdot e^{-t} \right) + \left(-t \cdot e^{-t} + \int 1 \cdot e^{-t} dt \right)$$

$$= -\frac{t^2 e^{-t}}{2} - t e^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1 \right) e^{-t}$$

$$= \left(-\frac{x^4}{2} - x^2 - 1 \right) e^{-x^2} + C$$

$$g(x) = -1 - x^2 - \frac{x^4}{2} + k e^{x^2}$$

for $k = 0$

$$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

13. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

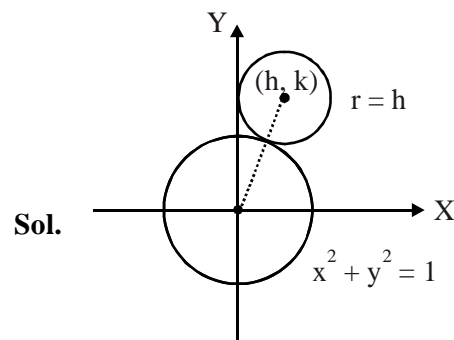
(1) $y = \sqrt{1+4x}$, $x \geq 0$

(2) $x = \sqrt{1+4y}$, $y \geq 0$

(3) $x = \sqrt{1+2y}$, $y \geq 0$

(4) $y = \sqrt{1+2x}$, $x \geq 0$

Official Ans. by NTA (4)



$$\sqrt{h^2 + k^2} = |h| + 1$$

$$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$$

$$\Rightarrow y^2 = 1 + 2x$$

$$\Rightarrow y = \sqrt{1+2x} ; x \geq 0.$$

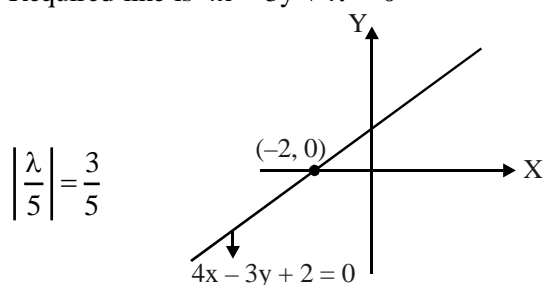
14. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin.

Then which one of the following points lies on any of these lines ?

- (1) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$
 (3) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$ (4) $\left(\frac{1}{4}, -\frac{1}{3}\right)$

Official Ans. by NTA (1)

Sol. Required line is $4x - 3y + \lambda = 0$



$$\Rightarrow \lambda = \pm 3.$$

So, required equation of line is $4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

$$(1) 4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$$

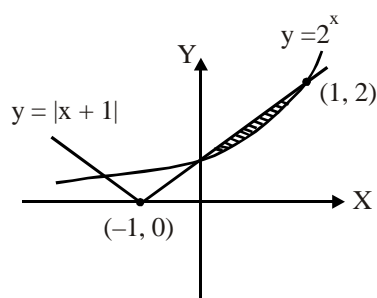
15. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

- (1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$
 (3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

Sol. Required Area

$$\int_0^1 ((x+1) - 2^x) dx$$



$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2} \right) - \left(0 + 0 - \frac{1}{\ln 2} \right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

16. If the plane $2x - y + 2z + 3 = 0$ has the distances

$$\frac{1}{3} \text{ and } \frac{2}{3} \text{ units from the planes } 4x - 2y + 4z + \lambda = 0$$

and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

- (1) 15 (2) 5
 (3) 13 (4) 9

Official Ans. by NTA (3)

Sol. $4x - 2y + 4z + 6 = 0$

$$\frac{|\lambda - 6|}{\sqrt{16 + 4 + 16}} = \frac{|\lambda - 6|}{6} = \frac{1}{3}$$

$$|\lambda - 6| = 2$$

$$\lambda = 8, 4$$

$$\frac{|\mu - 3|}{\sqrt{4 + 4 + 1}} = \frac{2}{3}$$

$$|\mu - 3| = 2$$

$$\mu = 5, 1$$

\therefore Maximum value of $(\mu + \lambda) = 13$.

17. If z and w are two complex numbers such that

$$|zw| = 1 \text{ and } \arg(z) - \arg(w) = \frac{\pi}{2}, \text{ then :}$$

- (1) $\bar{z}w = i$ (2) $\bar{z}w = -i$
 (3) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (4) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

Official Ans. by NTA (2)

Sol. $|z| \cdot |w| = 1$ $z = re^{i(\theta + \pi/2)}$ and $w = \frac{1}{r} e^{i\theta}$

$$\bar{z} \cdot w = e^{-i(\theta + \pi/2)} \cdot e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$z \cdot \bar{w} = e^{i(\theta + \pi/2)} \cdot e^{-i\theta} = e^{i(\pi/2)} = i$$

18. Let a, b and c be in G. P. with common ratio r, where

$a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

Official Ans. by NTA (2)

Sol. $b = ar$

$$c = ar^2$$

3a, 7b and 15c are in A.P.

$$\Rightarrow 14b = 3a + 15c$$

$$\Rightarrow 14(ar) = 3a + 15ar^2$$

$$\Rightarrow 14r = 3 + 15r^2$$

$$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r-1)(5r-3) = 0$$

$$r = \frac{1}{3}, \frac{3}{5}$$

Only acceptable value is $r = \frac{1}{3}$, because

$$r \in \left(0, \frac{1}{2}\right]$$

$$\therefore c, d = 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$$

$$\therefore 4^{\text{th}} \text{ term} = 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$$

19. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ equal to :

(1) $3^{7/6} - 3^{5/6}$

(2) $3^{5/3} - 3^{1/3}$

(3) $3^{4/3} - 3^{1/3}$

(4) $3^{5/6} - 3^{2/3}$

Official Ans. by NTA (1)

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x \cdot \sin x} dx$

$$= \int \frac{\tan^{2/3} x \cdot \sec^2 x \cdot dx}{\tan^2 x}$$

$$= \int \frac{\sec^2 x}{\tan^{4/3} x} \cdot dx \quad \{\tan x = t, \sec^2 x dx = dt\}$$

$$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3(t^{-1/3})$$

$$\Rightarrow I = -3 \tan(x)^{-1/3}$$

$$\Rightarrow I = \frac{3}{(\tan x)^{1/3}} \Bigg|_{\pi/6}^{\pi/3} = -3 \left[\frac{1}{(\sqrt{3})^{1/3}} - (\sqrt{3})^{1/3} \right]$$

$$= 3 \left(3^{1/3} - \frac{1}{3^{1/6}} \right) = 3^{7/6} - 3^{5/6}$$

20. Let $y = y(x)$ be the solution of the differential

equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$. Then :

(1) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) - y\left(\frac{-\pi}{4}\right) = \sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right) + y\left(\frac{-\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Official Ans. by NTA (2)

25. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to :}$$

- (1) 6 (2) 1
(3) 0 (4) -4

Official Ans. by NTA (3)

Sol. By expansion, we get
 $-5x^3 + 30x - 30 + 5x = 0$
 $\Rightarrow -5x^3 + 35x - 30 = 0$
 $\Rightarrow x^3 - 7x + 6 = 0$, All roots are real
 So, sum of roots = 0

26. Let $f(x) = \log_e(\sin x)$, ($0 < x < \pi$) and $g(x) = \sin^{-1}(e^{-x})$, ($x \geq 0$). If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

- (1) $a\alpha^2 - b\alpha - a = 0$
 (2) $a\alpha^2 + b\alpha - a = -2\alpha^2$
 (3) $a\alpha^2 + b\alpha + a = 0$
 (4) $a\alpha^2 - b\alpha - a = 1$

Official Ans. by NTA (4)

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$
 $(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

27. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in \mathbb{R}$,

($x \neq \pm\sqrt{3}$), at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

- (1) $|6\alpha + 2\beta| = 19$
 (2) $|2\alpha + 6\beta| = 11$
 (3) $|6\alpha + 2\beta| = 9$
 (4) $|2\alpha + 6\beta| = 19$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}\bigg|_{(\alpha, \beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given that :

$$\frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}$$

$$\Rightarrow \alpha = 0, \pm 3 \quad (\alpha \neq 0)$$

$$\Rightarrow \beta = \pm \frac{1}{2}. \quad (\beta \neq 0)$$

$$|6\alpha + 2\beta| = 19$$

28. The number of real roots of the equation

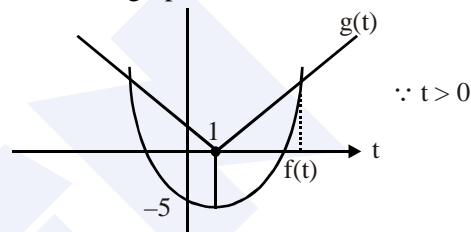
$$5 + |2^x - 1| = 2^x(2^x - 2) \text{ is :}$$

- (1) 2 (2) 3
(3) 4 (4) 1

Official Ans. by NTA (4)

Sol. Let $2^x = t$
 $5 + |t - 1| = t^2 - 2t$
 $\Rightarrow |t - 1| = (t^2 - 2t - 5)$
 $\quad \quad \quad g(t) \quad \quad \quad f(t)$

From the graph



So, number of real root is 1.

29. If $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to :-

- (1) -7 (2) -4
(3) 5 (4) 1

Official Ans. by NTA (1)

Sol. $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$

$$1 - a + b = 0 \quad \dots(i)$$

$$2 - a = 5 \quad \dots(ii)$$

$$\Rightarrow a + b = -7.$$

30. The negation of the boolean expression

$\sim s \vee (\sim r \wedge s)$ is equivalent to :

- (1) r (2) $s \wedge r$
(3) $s \vee r$ (4) $\sim s \wedge \sim r$

Official Ans. by NTA (2)

Sol. $\sim(\sim s \vee (\sim r \wedge s))$

$$s \wedge (r \vee \sim s)$$

$$(s \wedge r) \vee (s \wedge \sim s)$$

$$(s \wedge r) \vee (c)$$

$$(s \wedge r)$$